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A model for evaluating creative work outcomes at Czech Art Colleges

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Abstract. The Register of Artistic Performances is currently being developed in CZ that will contain information on works of art originating from creative activities of art colleges and faculties. Outcomes in various fields of artistic production will be divided into 27 categories, based on their significance, size, and international reception (each criterion classifies into three classes), and each category will be assigned a score. The total score will provide a basis for allocating a part of the state-budget subsidy among art colleges.

The paper discusses the model used to determine scores for each category. The approach is based on Saaty's method, which expertly compares significances of all 27 categories. Creating Saaty's matrix of preference intensities for abstract categories, while maintaining acceptable consistency for such a large matrix, is a difficult task. In the paper we describe a procedure for obtaining required information from a team of persons responsible for different fields of artistic production. A search for solution to this problem has led to new interpretations of Saaty's matrix elements and its consistency condition.

Keywords: Multiple criteria evaluation, Saaty's method, work of art.

JEL Classification: C44
AMS Classification: 91B74

1 Register of Artistic Performances, Classification of Works of Art

The Register of Artistic Performances (RAP) is currently being developed in the Czech Republic that should contain information on works of art originating from creative activities of art colleges and faculties (see [6]). The RAP is conceived as an analogy to the register of R&D outcomes where information on outcomes of research institutions (including universities) has been collected for some years already. In both the registers the outcomes are stored under several categories. These categories are assigned scores. The sum of scores of all the outcomes of a given university is considered an indicator of its performance in the area of creative activity. These numeric values can then be used in decisions regarding one part of the total money to be allocated among universities from the state budget.

The structure of the evaluated categories used in the Czech model was inspired, to some extent, by the artistic categories in the Slovak Republic (see [7]). However, the mathematical model used to determine scores for each category in Slovakia is quite different.

For the purposes of registration of works of art originating from creative activities of the Czech art colleges and faculties, the whole area of artistic production is divided into seven fields: fine arts, design, architecture, theatre, film, literature, and music.

Each piece of art, regardless of the field, is categorized according to the following three criteria:

- Relevance or significance of the piece;
- Extent of the piece;
- Institutional and media reception/impact of the piece.

In each criterion, three different levels are distinguished (denoted by capital letters for easier handling):

- The criterion *Relevance or significance of the piece*:
  - A – a new piece of art or a performance of crucial significance;
  - B – a new piece of art or a performance containing numerous important innovations;
  - C – a new piece of art or a performance pushing forward modern trends.

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- The criterion *Extent of the piece*:
  - K - a piece of art or a performance of large extent;
  - L - a piece of art or a performance of medium extent;
  - M - a piece of art or a performance of limited extent.
- The criterion *Institutional and media reception/impact of the piece*:
  - X – international reception/impact,
  - Y – national reception/impact,
  - Z – regional reception/impact.

The resulting category for a piece of art is given by a combination of three capital letters – e.g. AKX, BKY, or CLZ. There are 27 categories altogether. The decision concerning the relevance or significance of the piece (choice of A, B or C) rests upon expert assessment; the experts have at their disposal general definitions of each category and examples of works of art in each category – for all three levels of each criterion and for all 7 fields of artistic production – to assist them in the decision process. As for the extent of the piece (levels K, L, M), all the classes are specified for all the fields of art. As for the institutional and media reception/impact, lists of institutions corresponding to categories X, Y, Z are available for all fields.

Le us notice, there are interactions among the three mentioned criteria. The first one (expertly defined *Relevance or significance of the piece of art*) and the third one (*Institutional and media reception/impact of the piece*) partly overlap. That means, we are not allowed to set separately the weights of criteria and the scores of levels for each of them, and then calculate the scores of categories by means of the weighted average operation. It is necessary to set directly the scores of the categories that are described by the triples of criteria levels.

2 Determining scores for particular categories of artistic production

Saaty's method (see [2, 3, 4]) served as a basis for determination of scores for all 27 categories of artistic production. However obvious it was that this mathematical tool is the most appropriate for such a task, certain challenges concerning its use were also clearly apparent: (1) a difficulty for a team of experts to express preferences with respect to abstract categories; (2) a difficulty to reach acceptable consistency of Saaty's matrix under such a large number of categories; (3) a consensus within the group of experts (professional guarantors of particular fields of art). The proposed solution to these problems will be described in the following paragraphs.

Admittedly, expressing one's opinion on intensities of preferences with respect to abstract categories is difficult. Experts, professional guarantors of artistic fields, were first asked to provide specific (historical) examples of works of art in all categories in their field. (This step was also important to ensure, or to suggest modifications to ensure, that corresponding categories be really comparable in terms of evaluation across fields.) Next, professional guarantors of each field of art set their preferences concerning pairs of categories, while considering the representatives (examples) of each category to aid them in their decisions.

Although it was possible for each of these experts to express their preferences separately, and only then to derive the collective preferences (from the individual ones), we used a different approach. The collective preferences were set directly at a team meeting of experts. The reason was that art-college experts are not used to work with mathematical models and individual inputting of required data could prove difficult for them. Achieving consensus was also intentionally preferred over averaging different opinions.

Great effort was made to find the best way of converting expert preferences concerning the 27 categories of artistic production (represented in each field of art by specific examples) into a mathematical model in order to determine their scores. To facilitate the process of inputting required data by the experts and to achieve the necessary consistency of this input, the following two-step procedure was performed:

In the first step, we have determined the order of importance of the categories by the pairwise comparison method (see [2, 5]). This method employs a matrix of preferences and indifferences $P = \left\{ p_{i,j} \right\}_{i,j=1,\dots,27}$. For its elements it holds that:

$p_{i,j} = 1,$ if the i^th category is more important than the j^th category;

$p_{i,j} = 0,5,$ if the i^th category is equally important as the j^th category;

$p_{i,j} = 0,$ if the j^th category is more important than the i^th category.

It is sufficient for the experts to fill in the upper right triangle of the matrix, that is, the elements $p_{i,j}, i < j,$ as $p_{i,i} = 0,5$ and $p_{j,i} = 1 - p_{i,j}$. The row sums $D_i = \sum_{j=1}^{27} p_{i,j}, i = 1,\dots,27$, determine the order of importance of the categories (their quasi-ordering, transitive and complete relation, that can be described as a linear ordering of classes of indifferent elements). We need to verify consistency of the preferences in the sense of transitivity, that is, whether it holds that $p_{i,k} \geq \max\left\{p_{i,j}, p_{j,k}\right\}$ for all $i, j, k \in \{1,...,27\}$. If the matrix is not consistent, we make a minimum amount of changes necessary for it to become so. These changes are then consulted with the team of experts and if they are approved of, we can proceed. All the changes actually made while solving our problem are summarized in Tab 1.

	1 AKX	2 AKY	3 AKZ	4 ALX	5 ALY	6 ALZ	7 AMX	8 AMY	9 AMZ	10 BKX	11 BKY	12 BKZ	13 BLX	14 BLY	15 BLZ	16 BMX	17 BMY	18 BMZ	19 CKX	20 CKY	21 CKZ	22 CLX	23 CLY	24 CLZ	25 CMX	26 CMY	27 CMZ	Prefe-rence	Preference order	
1 AKX	0,5	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	26,5	1	AKX
2 AKY		0,5	0,5	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	25	2	AKY
3 AKZ			0,5	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	25	2	AKZ
4 ALX				0,5	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	23,5	4	ALX
5 ALY					0,5	1,0	0,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	21,5	6	ALY
6 ALZ						0,5	0,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	20,5	7	ALZ
7 AMX							0,5	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	22,5	5	AMX
8 AMY								0,5	1,0	0,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	18,5	9	AMY
9 AMZ									0,5	0,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	17,5	10	AMZ
10 BKX										0,5	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	19,5	8	BKX
11 BKY											0,5	1,0	0,5	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	16	11	BKY
12 BKZ												0,5	0,0	1,0	1,0	0,5	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	14	13	BKZ
13 BLX													0,5	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	16	11	BLX
14 BLY														0,5	1,0	0,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	12,5	15	BLY
15 BLZ															0,5	0,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	11,5	16	BLZ
16 BMX																0,5	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	14	13	BMX
17 BMY																	0,5	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	10,5	17	BMY
18 BMZ																		0,5	0,5	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	9	18	BMZ
19 CKX																			0,5	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	9	18	CKX
20 CKY																				0,5	1,0	0,0	1,0	1,0	1,0	1,0	1,0	6,5	21	CKY
21 CKZ																					0,5	0,0	1,0	1,0	1,0	1,0	1,0	5,5	22	CKZ
22 CLX																						0,5	1,0	1,0	1,0	1,0	1,0	7,5	20	CLX
23 CLY																							0,5	1,0	0,0	1,0	1,0	3,5	24	CLY
24 CLZ																								0,5	0,0	1,0	1,0	2,5	25	CLZ
25 CMX																									0,5	1,0	1,0	4,5	23	CMX
26 CMY																										0,5	1,0	1,5	26	CMY
27 CMZ																											0,5	0,5	27	CMZ

changes made to achieve consistency of the matrix resulting form the final order
change made from 0,5 to 1 or from 1 to 0,5 (**"small change"**)
change made from 0 to 1 ("**big change**")

Table 1 The pairwise comparison matrix with highlighted changes.

In the second step, Saaty's matrix of preference intensities $S = \left\{s_{i,j}\right\}_{i,j=1,...,27}$ was constructed for categories numbered in ascending order according to their significance determined in the previous step. Again, it was sufficient to fill in the upper right triangle of the matrix. The elements $s_{i,j}, i < j,$ were set as follows:

$s_{i,j} = 1 \ldots$ the i[th] and j[th] categories are **equally** important;

$s_{i,j} = 3 \ldots$ the i[th] category is **slightly more important** than the j[th] category;

$s_{i,j} = 5 \ldots$ the i[th] category is **strongly more important** than the j[th] category;

$s_{i,j} = 7 \ldots$ the i[th] category is **very strongly more important** than the j[th] category;

$s_{i,j} = 9 \ldots$ the i[th] category is **extremely more important** than the j[th] category.

It holds that $s_{i,i} = 1$ and $s_{j,i} = 1/s_{i,j}$, for the intensity of preference $s_{i,j}$ expresses the ratio of preferences between both categories.

The traditional requirement for consistency in Saaty's method, that is $s_{i,k} = s_{i,j} \cdot s_{j,k}$ for all $i, j, k \in \{1,...,27\}$, is basically unachievable. For example, consider only four arbitrary objects that are linearly ordered according to their importance. If each of them is just slightly more important than the following one, then in the case of full consistency the first one would have to be 27 times more important than the fourth. But the maximum value available for expressing intensity of preference is nine (as is shown by psychological research [3], this is the highest number of levels of importance that human is able to distinguish). We have weakened the original requirement on consistency, which was too strong, and for the purposes of our work we have requested $s_{i,k} \geq \max\left\{s_{i,j}, s_{j,k}\right\}$ for all $i, j, k \in \{1,...,27\}$. When the categories are numbered as to their importance, this requirement is easy to verify. In addition to the fact that the matrix S has to be reciprocal (i.e. $s_{i,i} = 1$ and $s_{i,j} = 1/s_{j,i}$ for $i, j \in \{1,...,27\}$) in view of the above-mentioned condition, consistency means that the elements of S are nondecreasing from left to right and from bottom up. If the matrix, as set by the experts, is not consistent, we propose the minimum amount of changes necessary for it to become so – the team of professional guarantors either approve of these changes or make their own to achieve consistency. Tab. 2 illustrates the changes actually made in our application in order to remove inconsistencies from the original matrix S. (Tab. 2 contains also changes induced by re-dividing the pairs of indifferent categories having originated from the pair-wise comparison method.)

		1 AKX	2 AKY	3 AKZ	4 ALX	5 AMX	6 ALY	7 ALZ	8 BKX	9 AMY	10 AMZ	11 BKY	12 BKZ	13 BLX	14 BMX	15 BLY	16 BLZ	17 BMY	18 BMZ	19 CKX	20 CLX	21 CKY	22 CKZ	23 CMX	24 CLY	25 CLZ	26 CMY	27 CMZ
1	AKX	1	5	5	5	5	5	5	5	5	5	5	5	5	5	5	7	7	9	9	9	9	9	9	9	9	9	9
2	AKY		1	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	7	7	7	7	9	9	9	9	9	9
3	AKZ			1	3	3	5	5	5	5	5	5	5	5	5	5	5	5	7	7	7	7	9	9	9	9	9	9
4	ALX				1	3	5	5	5	5	5	5	5	5	5	5	5	5	5	5	7	7	7	9	9	9	9	9
5	AMX					1	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	7	7	7	9	9	9
6	ALY						1	3	3	5	5	5	5	5	5	5	5	5	5	5	5	5	5	7	7	7	9	9
7	ALZ							1	3	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	7	7	7	9
8	BKX								1	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	7	7	7
9	AMY									1	3	5	5	5	5	5	5	5	5	5	5	5	5	5	5	7	7	7
10	AMZ										1	5	5	5	5	5	5	5	5	5	5	5	5	5	5	7	7	7
11	BKY											1	5	5	5	5	5	5	5	5	5	5	5	5	5	7	7	7
12	BKZ												1	3	5	5	5	5	5	5	5	5	5	5	5	7	7	7
13	BLX													1	5	5	5	5	5	5	5	5	5	5	5	7	7	7
14	BMX														1	3	3	3	5	5	5	5	5	5	5	7	7	7
15	BLY															1	3	3	5	5	5	5	5	5	5	7	7	7
16	BLZ																1	3	5	5	5	5	5	5	5	5	7	7
17	BMY																	1	5	5	5	5	5	5	5	5	7	7
18	BMZ																		1	5	5	5	5	5	5	5	5	5
19	CKX																			1	3	5	5	5	5	5	5	5
20	CLX																				1	5	5	5	5	5	5	5
21	CKY																					1	3	3	5	5	5	5
22	CKZ																						1	3	5	5	5	5
23	CMX																							1	5	5	5	5
24	CLY																								1	3	3	3
25	CLZ																									1	3	3
26	CMY																										1	3
27	CMZ																											1

3,5,7,9	změny vyplývající z rozdělení kategorií
3,5	změny respektující párové porovnávání sousedních kategorií
5	změny pro udržení konzistence vyvolané zadáním "červených hodnot"

Table 2 Saaty's matrix of preference intensities with highlighted changes.

Under the assumption that S is close enough to an ideally consistent matrix (i.e. matrix that fulfills $s_{i,k} = s_{i,j} \cdot s_{j,k}$ for all $i, j, k \in \{1, ..., 27\}$), the scores of 27 categories, representing their relative importance, are calculated by Saaty's method as components of the eigenvector corresponding to the largest eigenvalue.

The resulting scores of artistic categories can also be obtained from S in a different way. The columns of S can be interpreted as repeated measurements of the relative importances of the 27 categories. These measurements are performed by the team of experts who compare all the categories with the first one, than the second one, and so on until the 27^{th} one. From the point of view of mathematical statistics, these are compositional data, i.e. data bearing only relative information (see [1]). Information contained in this data can be expressed by estimating its mean value. A proper estimator of the mean value of this kind of data is a vector whose components are geometric means of the corresponding components of vectors representing single measurements. The relative scores of all 27 categories can be also obtained by computing geometric means of the rows of Saaty's matrix (this calculation method is known as the logarithmic least squares method, see [2]). This weaker consistency of S ($s_{i,k} \geq \max\{s_{i,j}, s_{j,k}\}$ for all $i, j, k \in \{1, ..., 27\}$) is then a natural requirement that allows for an easy check on consistency of the expertly entered data. The facts that S has to be reciprocal and, with the categories ordered according to their importance, that the values of a well entered matrix S must be nondecreasing from left to right and from bottom up can serve as a good guiding principle for teams of experts in defining the preference intensities of pairs of categories.

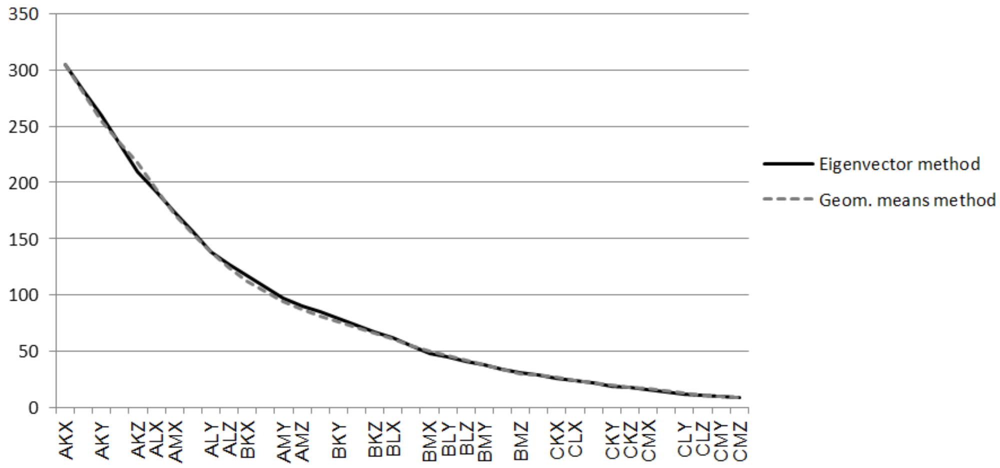

Figure 1 Graphical comparison of the eigenvector method with the geometric means method.

Category	Relevance or significance	Extent	Institutional reception	Eigenvector method	Geom. means method
AKX	Crucial significance and originality	Large	International	**305**	**305**
AKY	Crucial significance and originality	Large	National	**259**	**254**
AKZ	Crucial significance and originality	Large	Regional	**210**	**217**
ALX	Crucial significance and originality	Medium	International	**191**	**194**
AMX	Crucial significance and originality	Small	International	**174**	**171**
ALY	Crucial significance and originality	Medium	National	**138**	**138**
ALZ	Crucial significance and originality	Medium	Regional	**127**	**124**
BKX	Bearing many important inovations	Large	International	**117**	**112**
AMY	Crucial significance and originality	Small	National	**97**	**94**
AMZ	Crucial significance and originality	Small	Regional	**90**	**87**
BKY	Bearing many important inovations	Large	National	**79**	**75**
BKZ	Bearing many important inovations	Large	Regional	**66**	**66**
BLX	Bearing many important inovations	Medium	International	**62**	**61**
BMX	Bearing many important inovations	Small	International	**48**	**50**
BLY	Bearing many important inovations	Medium	National	**44**	**46**
BLZ	Bearing many important inovations	Medium	Regional	**40**	**41**
BMY	Bearing many important inovations	Small	National	**37**	**38**
BMZ	Bearing many important inovations	Small	Regional	**31**	**30**
CKX	Developing current trends	Large	International	**26**	**26**
CLX	Developing current trends	Medium	International	**24**	**24**
CKY	Developing current trends	Large	National	**19**	**20**
CKZ	Developing current trends	Large	Regional	**17**	**18**
CMX	Developing current trends	Small	International	**16**	**16**
CLY	Developing current trends	Medium	National	**12**	**13**
CLZ	Developing current trends	Medium	Regional	**10**	**11**
CMY	Developing current trends	Small	National	**9**	**9**
CMYŹ	Developing current trends	Small	Regional	**8**	**9**

Table 3 Scores obtained by the Saaty matrix eigenvector method and those determined as geometric means of rows of S.

Tab. 3 compares the scores determined by the Saaty matrix eigenvector method with those determined as geometric means of the rows. The scores are normalized so that the maximum is 305 (analogy to R&D outcomes evaluation). It is easy to see that the differences between these two methods are not significant, see Fig. 1. The Saaty matrix eigenvector method will be used in testing the model on the first real dataset, gathered by Czech art colleges and faculties for the years 2008 to 2010.

3 Conclusion

The Register of Artistic Performances and the methodology of evaluating artistic production originating from creative activities of art colleges and faculties are currently being pilot-tested in the Czech Republic. At present, our effort is focused on refining the triplets of class specification for all three criteria and for all the fields of art, and particularly on developing a most objective mechanism of expert classification of artistic production into 27 categories.

The mathematical model for score determination was developed in an effort to achieve the best possible conversion of preferences of the expert team into scores for different categories of artistic production. With Saaty's method serving as an appropriate basis, the solution to this problem required its implementation in a special procedure.

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